MODELING AND SIMULATION
OF A DOUBLE PENDULUM WITH PAD

- a draft version -

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Abstract

In this paper, results of the simulation of a double pendulum with a horizontal pad are presented. Pendulums are arranged in such a way that in the static equilibrium, small pendulum takes the vertical position, while the big pendulum is in a horizontal position and rests on the pad. Motion during one half oscillation is investigated. Impact of the big pendulum on the pad is considered to be ideally inelastic. Characteristic positions and angular velocities of both pendulums, as well as their energies at each instant of time are presented. Obtained results proved to be in accordance with the motion of the real physical system. Double pendulum with pad refers to the two-stage mechanical oscillator that is invented, patented and constructed by Serbian inventor Veljko Milković (www.veljkomilkovic.com).

Key words: double pendulum, nonlinear oscillations, impact

1. Introduction

Double pendulum is a mechanical system that is most widely used for demonstration of the chaotic motion. It is described with two highly coupled, nonlinear, 2nd order ODE's which makes is very sensitive to the initial conditions. Although its motion is deterministic in nature, sensitivity to initial conditions makes its motion unpredictable or 'chaotic' in the long turn.

Double pendulum with a pad that constraints motion of the big pendulum is a mechanical system which is not analyzed in the literature. It is consisted of a small pendulum in a vertical position, connected to the big pendulum which takes horizontal position and rests on the pad in the state of static equilibrium. When the small pendulum is excited to oscillate, big pendulum is lifted and moves to the maximum point and then goes back to original position where hits the pad. Experiments with the real system seemingly showed that energy produced by the impact of the big pendulum is somehow larger than the energy required for maintaining the oscillations. It is well known that this is not possible in the gravitational field, where conservative forces act. Therefore, model for this system was developed and simulated in order to show the discrepancy in motion predicted by simulation and the real motion.

2. Motion description

Double pendulum with pad is shown in Fig.1. System consists of: (i) the big pendulum (K_2) which can rotate around its axis (O_2) attached to the construction support, (ii) the small pendulum (K_2) with its axis on the big pendulum (O_1) and (iii) the horizontal pad. In the state of static equilibrium, small pendulum takes the vertical position ($\theta_1 = 0$), and big pendulum takes the horizontal position ($\theta_2 = \dfrac{\pi}{2}$, Fig. 1a) and rest on the pad.

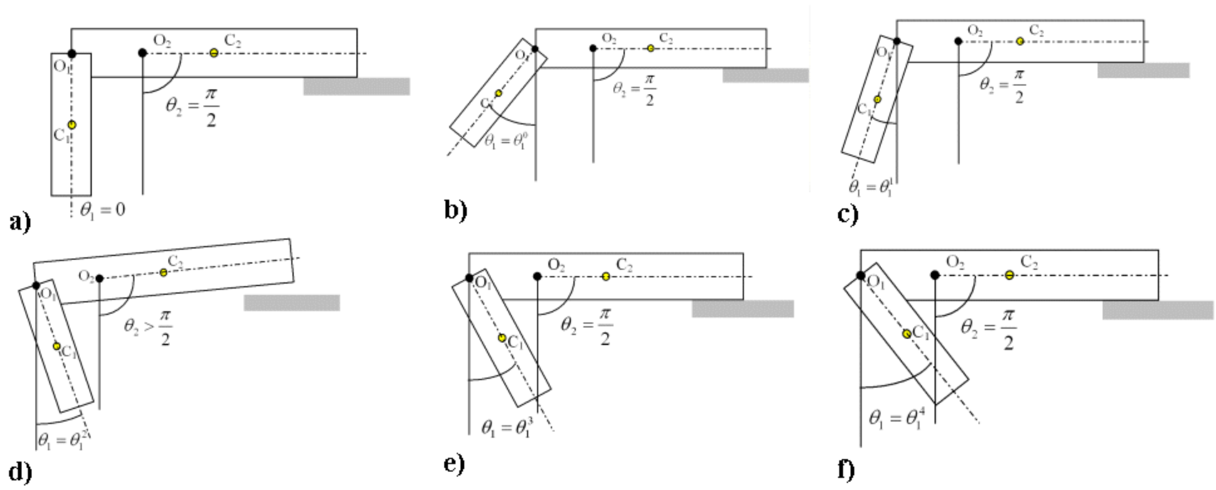

Figure 1. Characteristic positions of the double pendulum with pad during one half oscillation

When small pendulum is taken out of the equilibrium position and released ($\theta_1 = \theta_1^0 < 0$, Fig.1b) it starts its motion toward equilibrium position. Angle θ_1 is being increased (decreases in its absolute value) so that vertical component of the small pendulum weight ($G_{1,v}$, Fig.2) increases. At the same time, vertical component of the centrifugal force, $F_{c,v}$, is increased. At the moment t_1, sum of these two forces gives momentum $M_{O_2}^{F_1}$ that is equal to the momentum of the big pendulum $M_{O_2}^{G_2}$. In that instant, big pendulum is lifted of the pad (Fig. 1c) and is being moved upward. After $\theta_1 = 0$ is reached, vertical forces of the small pendulum are decreasing and big pendulum slows down reaching the maximum position in the moment t_2 (Fig.1d). Big pendulum starts moving downward while small pendulum is slowing down. At the moment t_3, big pendulum hits the pad (Fig.1e) and we assume ideally inelastic impact there. At the moment t_4, small pendulum reaches maximum position and stops (Fig. 1f). Period from t_0 to t_4 is called one half-oscillation.

3. Mathematical models

One half-oscillation period can be divided into three characteristic periods: (i) t_0 to t_1, only small pendulum is in motion, (ii) t_1 to t_3, the whole system is in motion and (iii) t_3 do t_4, only small pendulum is in motion. Mathematical model of a single pendulum is needed for periods (i) and (iii), double pendulum model is required for period (ii). It is also necessary to know time instants t_1 and t_3 when motion is switched from single to double pendulum and vice versa. From the condition that momentum of forces are equal $M_{O_2}^{F_1} = M_{O_2}^{G_2}$, time t_1 is determined. Condition $\theta_2 = \pi/2 - \alpha$ (Fig.3) determines time of the impact t_3. It is also necessary to determine the change in angular velocity of the small pendulum due to the impact.

Time t_4 , when the small pendulum stops is calculated from the condition that angular velocity of the small pendulum is equal to zero.

3.1 Physical pendulum model

Mathematical model of the physical pendulum is represented by the 2nd order ordinary differential equation,

$$\ddot{\theta}_1 + \frac{M_1 \cdot g \cdot b}{J_{O_1}} \cdot \sin(\theta_1) = 0 \tag{1}$$

with initial conditions,

$$\theta_1(0) = \theta_1^0, \quad \dot{\theta}_1(0) = \dot{\theta}_1^0 \tag{1a}$$

where M_1 is pendulum mass, b is $\overline{O_1C_1}$ and J_{O_1} is moment of inertia with respect to axis O_1.

This 2nd order system can be readily transformed into the system of two first order ODE's. Making use of the substitution,

$$v_0 = \theta_1, \quad v_1 = \dot{\theta}_1 \tag{2}$$

we get ODE system with the new initial conditions,

$$\frac{dv_0}{dt} = v_1 \tag{3}$$

$$\frac{dv_1}{dt} = -\frac{M_1 g b}{J_{O_1}} \sin(v_0) \tag{3a}$$

$$v_0(0) = \theta_1^0, \; v_1(0) = \dot{\theta}_1^0 \tag{3b}$$

3.2 Force momentums

Degree of generality to which analysis is confined to is presented in Fig.2. In general case, point O_2 is out of the big pendulum axis of symmetry, and point O_2 doesn't belong to the line O_2C_2. These generalities allow investigation of influence of positions of points O_1 and O_2 on the characteristics of the system. Limitation that point O_1 lies on the axis of symmetry is kept since it is not of great importance in this analysis.

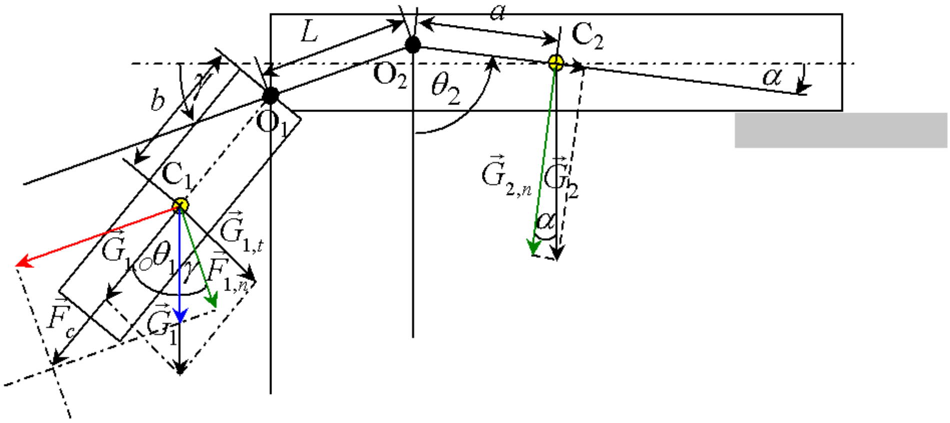

Figure 2. Forces at the moment when big pendulum is lifted of the pad

Characteristic points with forces acting upon them are presented in Fig. 2. It is needed to know the moments in point O_1 relative to point O_2 and moment in the point C_2 relative to the point O_2. Weight of the small pendulum $\vec{G}_1$ can be represented with component $\vec{G}_{1,O}$ along the O_1C_1 axis and component $\vec{G}_{1,t}$ that is tangential to the curved path ($G_{1,O} = G_1 \cdot \cos(\theta_1)$, $G_{1,t} = G_1 \cdot \sin(\theta_1)$). Centrifugal force that act along the axis O_1C_1 is $F_c = M_1 \cdot \dot{\theta}_1^{\,2} \cdot b$ and is summed up with force $G_{1,O}$ to give total force at the point O_1, $F_1 = F_c + G_{1,O}$. Moment of the force F1 with respect to axis O2 is,

$$M_{O_2}^{F_1} = F_1 \cdot \cos(\theta_1 - \gamma) \cdot L \text{ (since it is } \theta_1 < 0, \gamma > 0\text{)}, \tag{4}$$

and force momentum of the big pendulum with respect to O2 is,

$$M_{O_2}^{G_2} = G_2 \cdot \cos(\alpha) \cdot a \tag{5}$$

So, change in the motion regime from small pendulum to double pendulum occurs when the following condition is satisfied,

$$F_1 = M_2 g \cdot \frac{\cos(\alpha)}{\cos(\theta_1 - \gamma)} \cdot \frac{a}{L} \tag{6}$$

Orientation of angles alfa and gama is denoted on the figure. Relations (4) and (5) are indepenent of the θ_1, α or γ signs.

3.3 Double pendulum model

Model for the double pendulum is needed for the simulation during the second part of the half-oscillation period. For the sake of simplicity, Fig.3 contains only characteristic points of both pendulums.

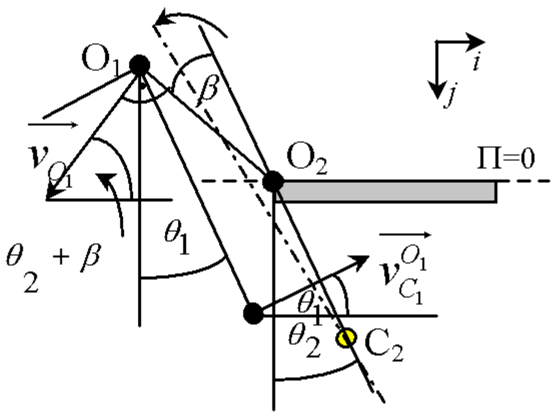

Figure 3. Characteristic points of the double pendulum with velocity vectors. Dash line represents the axis of the big pendulum

Lagrange equations (accumulation term equals to zero) are,

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_1} - \frac{\partial L}{\partial \theta_1} = 0 \tag{7}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_2} - \frac{\partial L}{\partial \theta_2} = 0 \tag{8}$$

Lagrangian is,

$$L = E_K - \Pi \tag{9}$$

Horizontal line through the point O_2 is accepted as reference level for the potential energy. Potential energy of the system is sum of potential energies of both pendulums,

$$\Pi = \underbrace{M_1 g(L\cos(\theta_2+\beta) - b\cos(\theta_1))}_{\Pi_1} - \underbrace{M_2 g a\cos(\theta_2)}_{\Pi_2} \tag{10}$$

Kinetic energy of the system is,

$$E_K = \underbrace{\frac{1}{2}M_1 v_{C_1}^2 + \frac{1}{2}J_{C_1}\omega_1^2}_{Ek_1} + \underbrace{\frac{1}{2}J_{O_2}\omega_2^2}_{Ek_2} \tag{11}$$

where J_{C_1} is the small pendulum moment of inertia with relation to point C_1 and J_{O_2} is the big pendulum moment of inertia with respect to O_2. Velocity at the point C_1 is the sum of two velocities $\overrightarrow{v_{C_1}} = \overrightarrow{v_{O_1}} + \overrightarrow{v_{C_1}^{O_1}}$,

$$\overrightarrow{v_{C_1}} = \underbrace{-v_{O_1}\cos(\theta_2+\beta)\cdot\vec{i} + v_{O_1}\sin(\theta_2+\beta)\cdot\vec{j}}_{v_{O_1}} + \underbrace{v_{C_1}^{O_1}\cos(\theta_1)\cdot\vec{i} - v_{C_1}^{O_1}\sin(\theta_1)\cdot\vec{j}}_{v_{C_1}^{O_1}} . \tag{12}$$

After introducing $v_{O_1} = L\dot{\theta}_2$ and $v_{C_1}^{O_1} = b\dot{\theta}_1$ we get,

$$\overrightarrow{v_{C_1}} = \vec{i}\cdot(-L\dot{\theta}_2\cos(\theta_2+\beta) + b\dot{\theta}_1\cos(\theta_1)) + \vec{j}\cdot(L\dot{\theta}_2\sin(\theta_2+\beta) - b\dot{\theta}_1\sin(\theta_1)) \tag{13}$$

$$\begin{aligned} v_{C_1}^2 &= L^2\dot{\theta}_2^2\cos^2(\theta_2+\beta) + b^2\dot{\theta}_1^2\cos^2(\theta_1) - 2Lb\dot{\theta}_1\dot{\theta}_2\cos(\theta_1)\cos(\theta_2+\beta) + \\ &+ L^2\dot{\theta}_2^2\sin^2(\theta_2+\beta) + b^2\dot{\theta}_1^2\sin^2(\theta_1) - 2Lb\dot{\theta}_1\dot{\theta}_2\sin(\theta_1)\sin(\theta_2+\beta) \end{aligned} \tag{14}$$

$$v_{C_1}^2 = L^2\dot{\theta}_2^2 + b^2\dot{\theta}_1^2 - 2Lb\dot{\theta}_1\dot{\theta}_2\cos(\theta_1 - (\theta_2+\beta)) \tag{15}$$

Using relations $\omega_1 = \dot{\theta}_1$, $\omega_2 = \dot{\theta}_2$ and the expression for moment of inertia $J_{O_2} = J_{C_2} + M_2 a^2$ we get the expression for the kinetic energy,

$$E_K = \frac{1}{2}M_1(L^2\dot{\theta}_2^2 + b^2\dot{\theta}_1^2 - 2Lb\dot{\theta}_1\dot{\theta}_2\cos(\theta_1 - (\theta_2+\beta))) + \frac{1}{2}J_{C_1}\dot{\theta}_1^2 + \frac{1}{2}(J_{C_2} + M_2 a^2)\dot{\theta}_2^2 \tag{16}$$

After introducing expressions for *Ek* i Π in Lagrangian, we have,

$$\begin{aligned} L = \frac{1}{2}\big[\dot{\theta}_1^2(M_1 b + J_{C_1}) + \dot{\theta}_2^2(L^2 M_1 + J_{C_2} + M_2 a^2) - 2M_1 Lb\dot{\theta}_1\dot{\theta}_2\cos(\theta_1 - (\theta_2+\beta)\big] - \\ - M_1 g(L\cos(\theta_2+\beta) - b\cos(\theta_1)) + M_2 ga\cos(\theta_2) \end{aligned} \tag{17}$$

Partial derivatives in the Lagrangian are,

$$\frac{\partial L}{\partial\dot{\theta}_1} = \dot{\theta}_1(M_1 b^2 + J_{C_1}) - LbM_1\dot{\theta}_2\cos(\theta_1 - (\theta_2+\beta)) \tag{18}$$

$$\frac{\partial L}{\partial\dot{\theta}_2} = \dot{\theta}_2(J_{C_2} + M_1 L^2 + M_1 a^2) - LbM_1\dot{\theta}_1\cos(\theta_1 - (\theta_2+\beta)) \tag{19}$$

$$\frac{\partial L}{\partial\theta_1} = M_1 Lb\dot{\theta}_1\dot{\theta}_2\sin(\theta_1 - (\theta_2+\beta) - M_1 gb\sin(\theta_1) \tag{20}$$

$$\frac{\partial L}{\partial\theta_2} = -M_1 Lb\dot{\theta}_1\dot{\theta}_2\sin(\theta_1 - (\theta_2+\beta)) + M_1 gL\sin(\theta_2+\beta) - M_2 ga\sin(\theta_2) \tag{21}$$

$$\begin{aligned} \frac{d}{dt}\frac{\partial L}{\partial\dot{\theta}_1} &= \ddot{\theta}_1(M_1 b^2 + J_{C_1}) - \ddot{\theta}_2 M_1 Lb\cos(\theta_1 - (\theta_2+\beta)) + \dot{\theta}_2 M_1 Lb\sin(\theta_1 - (\theta_2+\beta))(\dot{\theta}_1 - \dot{\theta}_2) = \\ &= \ddot{\theta}_1(M_1 b^2 + J_{C_1}) - \ddot{\theta}_2 M_1 Lb\cos(\theta_1 - (\theta_2+\beta)) + \dot{\theta}_1\dot{\theta}_2 M_1 Lb\sin(\theta_1 - (\theta_2+\beta)) - \dot{\theta}_2^2 M_1 Lb\sin(\theta_1 - (\theta_2+\beta)) \end{aligned} \tag{22}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_2} = \ddot{\theta}_2(J_{C_2} + M_1L^2 + M_1a^2) - \ddot{\theta}_1 M_1 Lb\cos(\theta_1 - (\theta_2 + \beta)) + M_1 Lb\dot{\theta}_1 \sin(\theta_1 - (\theta_2 + \beta))(\dot{\theta}_1 - \dot{\theta}_2) =$$
$$= \ddot{\theta}_2(J_{C_2} + M_1L^2 + M_1a^2) - \ddot{\theta}_1 M_1 Lb\cos(\theta_1 - (\theta_2 + \beta)) + \dot{\theta}_1^2 M_1 Lb\sin(\theta_1 - (\theta_2 + \beta)) - \dot{\theta}_1\dot{\theta}_2 M_1 Lb\sin(\theta_1 - (\theta_2 + \beta)) \tag{23}$$

After using these expressions in the Lagrange equations, we get,

$$\ddot{\theta}_1(\underbrace{J_{C_1} + M_1b^2}_{A}) - \ddot{\theta}_2\underbrace{M_1Lb}_{B}\cos(\theta_1 - (\theta_2 + \beta)) - \dot{\theta}_2^2\underbrace{M_1Lb}_{B}\sin(\theta_1 - (\theta_2 + \beta)) + \underbrace{M_1gb}_{C}\sin(\theta_1) = 0 \tag{24}$$

$$\ddot{\theta}_2(\underbrace{J_{C_1} + M_1L^2 + M_2a^2}_{D}) - \ddot{\theta}_1\underbrace{M_1Lb}_{B}\cos(\theta_1 - (\theta_2 + \beta)) + \dot{\theta}_1^2\underbrace{M_1Lb}_{B}\sin(\theta_1 - (\theta_2 + \beta)) + \underbrace{M_2ga}_{E}\sin(\theta_2) - \underbrace{M_1gL}_{F}\sin(\theta_2 + \beta) \tag{25}$$

Using the following expressions,

$$\begin{gathered} A = J_{C_1} + M_1b^2;\;\; B = M_1Lb;\;\; C = M_1gb \\ D = J_{C_1} + M_1L^2 + M_2a^2;\;\; E = M_2ga;\;\; F = M_1gl \end{gathered} \tag{26}$$

equations become more readable,

$$\ddot{\theta}_1 = \frac{B\cos(\theta_1 - (\theta_2 + \beta))}{A}\ddot{\theta}_2 + \frac{B\sin(\theta_1 - (\theta_2 + \beta))}{A}\dot{\theta}_2^2 - \frac{C\sin(\theta_1)}{A} \tag{27}$$

$$\ddot{\theta}_2 = \frac{B\cos(\theta_1 - (\theta_2 + \beta))}{D}\ddot{\theta}_1 - \frac{B\sin(\theta_1 - (\theta_2 + \beta))}{D}\dot{\theta}_1^2 + \frac{F\sin(\theta_2 + \beta)}{D} - \frac{E\sin(\theta_2)}{D} \tag{28}$$

In terms of angular accelerations ($\ddot{\theta}_1$ i $\ddot{\theta}_2$), these equations are linear and can be linearly combined in the equations where each one of them shows up independently. Substituting second equation into the first and first equation into the second, the following system is obtained,

$$\ddot{\theta}_1 = \frac{1}{A - \frac{B^2}{D}\cos^2(\theta_1 - (\theta_2 + \beta))}\begin{bmatrix} -\frac{B^2}{D}\frac{\sin(2(\theta_1 - (\theta_2 + \beta)))}{2}\dot{\theta}_1^2 + B\sin(\theta_1 - (\theta_2 + \beta))\cdot\dot{\theta}_2^2 - \ldots \\ -\frac{EB}{D}\sin(\theta_2)\cos(\theta_1 - (\theta_2 + \beta)) + \frac{FB}{D}\sin(\theta_2 + \beta)\cos(\theta_1 - (\theta_2 + \beta)) - C\sin(\theta_1) \end{bmatrix} \tag{29}$$

$$\ddot{\theta}_2 = \frac{1}{D - \frac{B^2}{A}\cos^2(\theta_1 - (\theta_2 + \beta))}\begin{bmatrix} \frac{B^2}{A}\frac{\sin(2(\theta_1 - (\theta_2 + \beta)))}{2}\dot{\theta}_2^2 - B\sin(\theta_1 - (\theta_2 + \beta))\cdot\dot{\theta}_1^2 - \ldots \\ -E\sin(\theta_2) + F\sin(\theta_2 + \beta) - \frac{CB}{A}\sin(\theta_1)\cos(\theta_1 - (\theta_2 + \beta)) \end{bmatrix} \tag{30}$$

Using these expressions,

$$u_0 = \theta_1,\; u_1 = \dot{\theta}_1,\; u_2 = \theta_2,\; u_3 = \dot{\theta}_2 \tag{31}$$

we get the equivalent ODE system,

$$\frac{du_0}{dt} = u_1 \tag{32}$$

$$\frac{du_1}{dt} = \frac{1}{A - \frac{B^2}{D}\cos^2(u_0 - (u_2 + \beta))} \begin{bmatrix} -\frac{B^2}{D}\frac{\sin(2(u_0 - (u_2+\beta)))}{2}u_1^2 + B\sin(u_0 - (u_2+\beta))\cdot(u_3)^2 - ... \\ -\frac{EB}{D}\sin(\theta_2)\cos(\theta_1 - (\theta_2+\beta)) + \frac{FB}{D}\sin(\theta_2+\beta)\cos(\theta_1 - (\theta_2+\beta)) - ... \\ -C\sin(\theta_1) \end{bmatrix} \tag{33}$$

$$\frac{du_2}{dt} = u_3 \tag{34}$$

$$\frac{du_3}{dt} = \frac{1}{D - \frac{B^2}{A}\cos^2(u_0 - (u_2 + \beta))} \begin{bmatrix} \frac{B^2}{A}\frac{\sin(2(u_0 - (u_2+\beta)))}{2}u_3^2 - B\sin(u_0 - (u_2+\beta))\cdot u_1^2 - ... \\ -E\sin(u_2) + F\sin(u_2+\beta) - \frac{CB}{A}\sin(u_0)\cos(u_0 - (u_2+\beta)) \end{bmatrix} \tag{35}$$

$$u_0(0) = \theta_1^0,\ u_1(0) = \dot{\theta}_1^0,\ \frac{du_1}{dt}(0) = \ddot{\theta}_1^0,\ u_2(0) = \theta_2^0,\ u_3(0) = \dot{\theta}_2^0,\ \frac{du_3}{dt}(0) = \ddot{\theta}_2^0 \tag{36}$$

3.4 Impact

It is assumed that the impact on the pad is ideally inelastic. This is in accordance with intention to determine the maximum energy that can be transferred to the pad with the impact. Until the impact, velocity of the small pendulum is vector sum of two velocities $\overrightarrow{v_{O_1}}$ and $\overrightarrow{v_{C_1}^{O_1}}$. At the moment of impact, velocity of the point O_1 becomes equal to zero, and one part of he velocity $\overrightarrow{v_{O_1}}$ which is $v_{add} = v_{O_1}\cos(\delta)$ becomes additional angular velocity of the small pendulum $\omega_{1,add} = \frac{v_{add}}{b}$. The other part of that velocity, $v_{axis} = v_{O_1}\sin(\delta)$, is transferred to the energy of impact in the point O_1 with the intensity $E_{axis} = \frac{1}{2}M_1 v_{axis}{}^2$. Although one part of this energy is lost, other part which equals to $E_{trans} = \frac{1}{2}M_1\left(v_{O_1}\sin^2(\delta)\right)^2$ is transferred to the pad. Angle δ= $f(\theta_1, \gamma)$ determines what part of the velocity will be transferred into $\omega_{1,add}$ at the moment of impact. Fig.4a-c show double pendulum in the moment of impact for all four combinations of the angles θ_1 i γ.

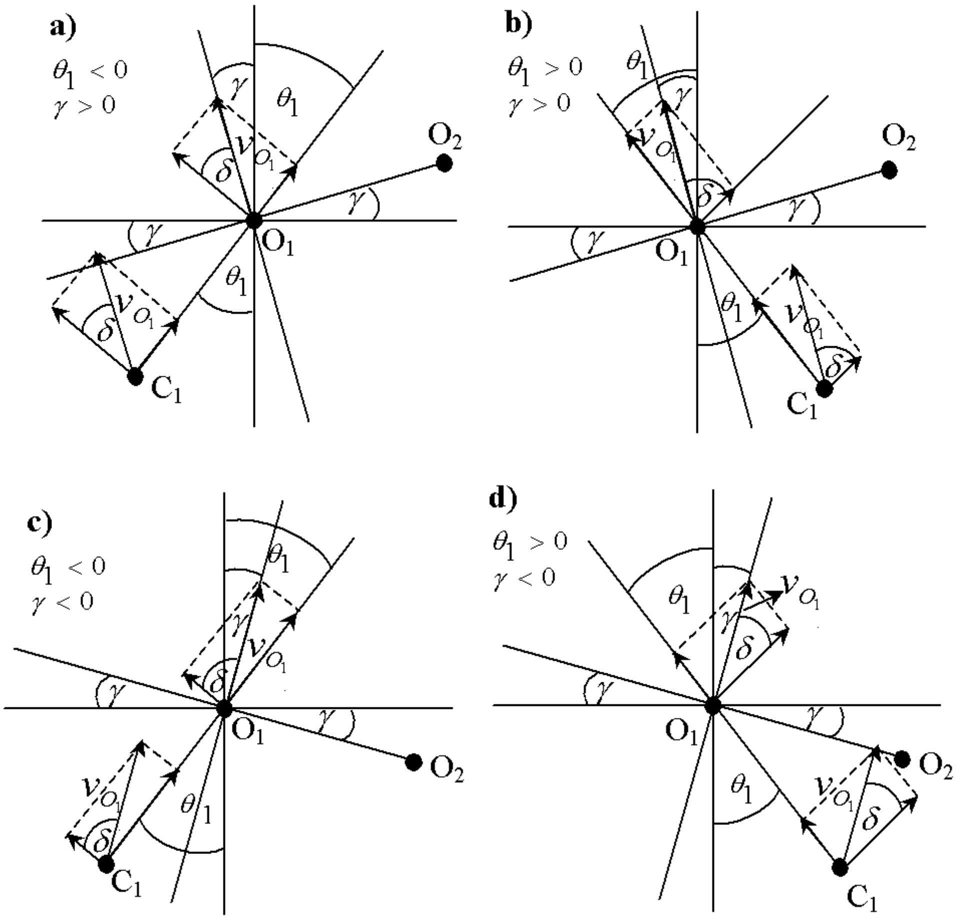

Figure 4. Angle δ at the moment of impact

For each of the cases presented in Fig.4, angle δ is separately determined. Results are presented in Table 1.

Table 1. Expressions for δ

No.	Angles θ_1, γ	Angle δ	Expression
a)	$\theta_1 < 0, \gamma > 0$	$\delta = \frac{\pi}{2} - \|\theta_1\| - \|\gamma\|$	$\delta = \frac{\pi}{2} + \theta_1 - \gamma$
b)	$\theta_1 > 0, \gamma > 0$	$\delta = \frac{\pi}{2} - \|\theta_1\| + \|\gamma\|$	$\delta = \frac{\pi}{2} - \theta_1 + \gamma$
c)	$\theta_1 < 0, \gamma < 0$	$\delta = \frac{\pi}{2} - \|\theta_1\| + \|\gamma\|$	$\delta = \frac{\pi}{2} + \theta_1 - \gamma$
d)	$\theta_1 > 0, \gamma < 0$	$\delta = \frac{\pi}{2} - \|\theta_1\| - \|\gamma\|$	$\delta = \frac{\pi}{2} - \theta_1 + \gamma$

4. Numerical procedure

General description of the numerical procedure follows. Code is given in the appendix.

1) Integrate ODE system for the small pendulum until the condition (6) is satisfied. It is equivalent to the condition that vertical component of the force F_1 is $F_{1,v} = M_2 g \cdot \frac{\cos(\alpha)}{\cos(\gamma)} \cdot \frac{a}{L}$. This condition is used for determination of the time instant t_1. Initial guess $t_1^0 = 0$ ensures convergence.
2) Integrate ODE system for the double pendulum until big pendulum hits the pad. Time instant t3 is determined from the condition $\theta_1 = \frac{\pi}{2} - \alpha$.
3) Determine new initial conditions for the small pendulum. Calculate energy of impact as kinetic energy of the big pendulum plus transferred energy E_{trans}.
4) Integrate ODE system of the small pendulum until time t_4 that is determined from the condition $\omega_1 = 0$.

Runge Kutta method of the 4th order with adaptive step size is used for integration of the ODE system. Nonlinear equations were solved using Newton's method.

5. Simulation results and discussion

Dimensions of the pendulums are given in Table 2. Material is iron of density $\rho = 7860 kg/m^3$.

Table 2. Pendulum dimensions, moments of inertia

	x [m]	y [m]	z [m]	Mass [kg]	J_C [kg·m^2]
K_1	0.3	0.08	0.05	7.546	0.061
K_2	0.6	0.1	0.05	23.58	0.727

Initial conditions used are $\theta_{1,0} = -60^\circ$, $\theta_{2,0} = 90^\circ$, $\omega_{1,0} = \omega_{2,0} = 0$, $\alpha = 0^\circ$, $\gamma = 0^\circ$

Table 3. Results

t_{max}	E_{impact}	$E_{tot,before}$	$E_{tot,after}$
0.488s	0.123J	-9.253J	-9.308J

One half-oscillation takes around 0.5s.

It seems that duration of the half-oscillation, required initial condition for the small pendulum (-60deg) and amplitude of the big pendulum are in quite good accordance with the behavior of the real, though a bit different physical system (Figs.5 and 6).

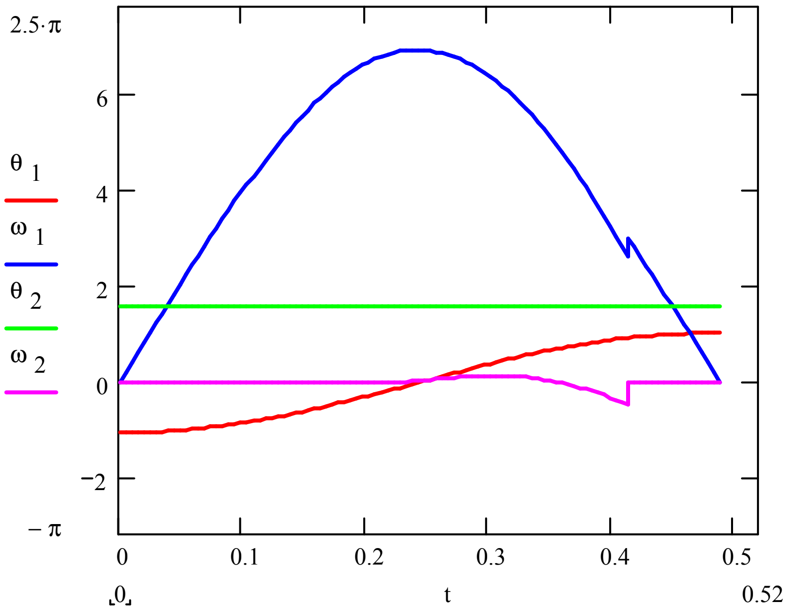

Figure 5. Positions and velocities of the pendulums

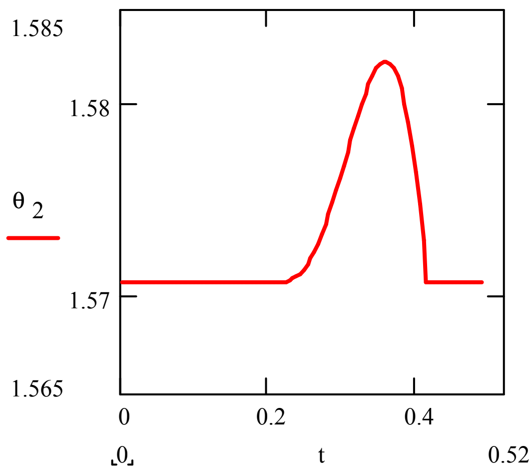

Figure 6. Closer look at the big pendulum behavior

Total energy of the double pendulum after the impact is lower than the total energy before the impact (-9.308<-9.253) which is reasonable (Fig.7). But when the transferred energy of impact is added (0.123J), the sum exceeds total initial energy (-9.253J). This is probably due to the incorrect calculation of the angular velocity $\omega_{1,add}$, so further investigation of this term should be the next step in validating this simulation.

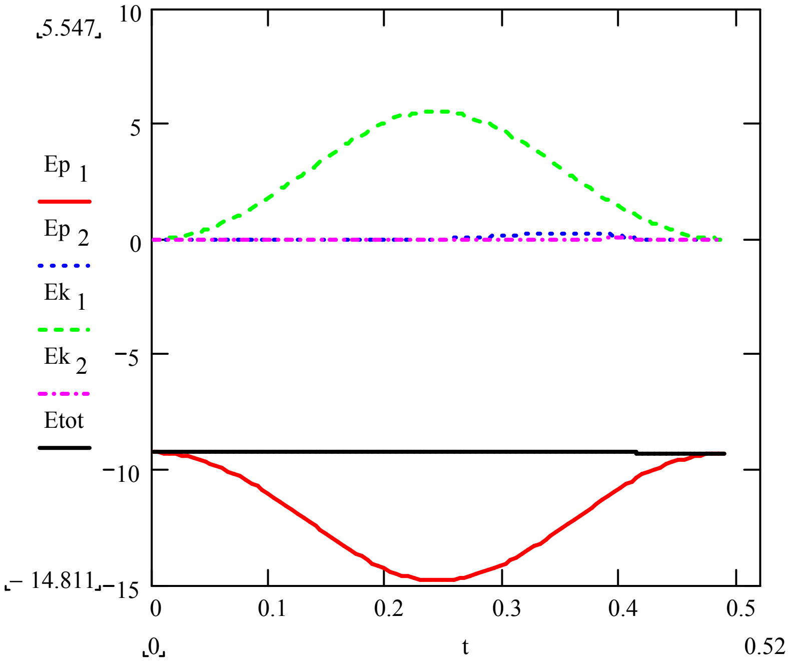

Figure 7. Potential, kinetic and total energy

6. Conclusions

Model for the double pendulum with pad was developed and simulated. Simulation results look reasonable and similar to the behavior of the real system. However, jump in the total energy after the impact has to be reconsidered and velocity that is added to the small pendulum after the impact must be analyzed further.

7. Acknowledgments

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