DYNAMICS OF A NONLINEAR MECHANICAL SYSTEM CONTAINING PENDULUMS

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Serbian inventor Veljko Milkovic [1] has introduced a continuous nonlinear mechanical system shown in Figure 1. Theoretical and experimental analyses reported by him and other authors in [1-2] suggest that the efficiency of this system in some excitation conditions can be surprisingly high. In the present study verification of these results is provided. The motion of the system is examined in the case of internal parametric resonance – case, in which some “anomalous” effects may occur.

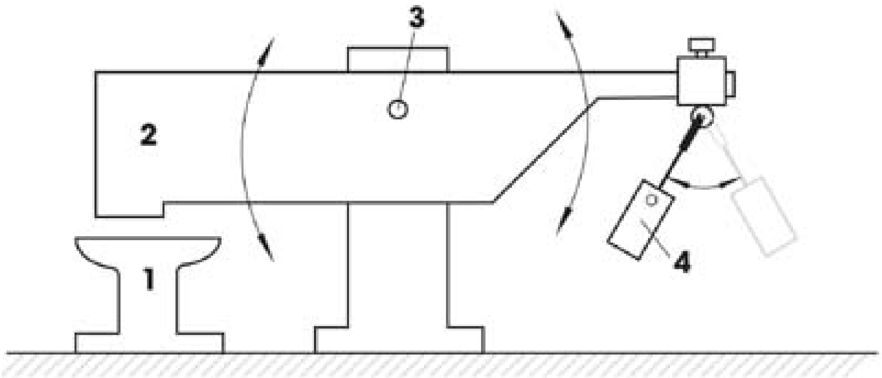

Figure 1. Mechanical hammer with a pendulum. 1 - anvil; 2 - massive lever; 3 - level axel; 4 - physical pendulum

To solve the problem, dimension reduction of this nonlinear dynamical system has been applied and the model, which consists of two coupled mathematical pendulums as shown in Figure 2, has been considered. A harmonic load is applied to one of the pendulums. The deflection of one pendulum and the angle of rotation of another one are introduced as the generalized coordinates. The motion of the system is examined for excitation close to the eigenfrequency of one of the pendulums in the case of internal parametric resonance (the eigenfrequency of one pendulum is twice as large as the eigenfrequency of another). The method of multiple scales in the form [3-4] is used.

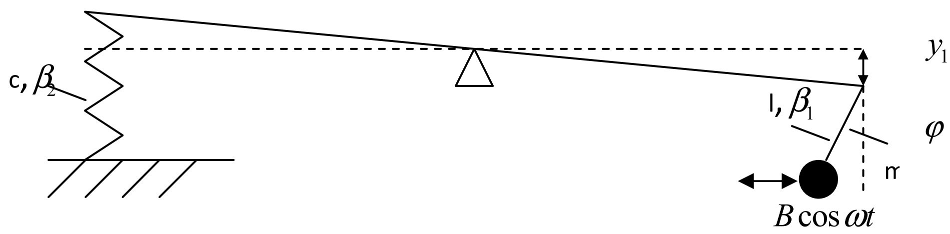

Figure 2. The model system.

The equations of motions, obtained using Lagrange's equations, have the form:

$$ml^2\ddot{\varphi} + \beta_1\dot{\varphi} + ml(g - \ddot{y}_1)\varphi = B\cos\omega t \quad (1)$$

$$m\ddot{y}_1 + \beta_2\dot{y}_1 + cy_1 - ml(\varphi\ddot{\varphi} + \dot{\varphi}^2) = mg \quad (2)$$

Here y_1 – the deflection of one pendulum, φ – the angle of rotation of another one. Dots designate differentiation with respect to time.

For the solution of problem, equations of motion are transformed to the non-dimensional form with the following scaling: $A = \frac{B}{ml^2}$, $2n_1 = \frac{\beta_1}{ml^2}$, $\lambda^2 = \frac{g}{l}$, $2n_2 = \frac{\beta_2}{m}$, $q^2 = \frac{c}{m}$. Introducing the new variable, $y = \frac{y_1}{l} - \frac{g}{lq^2}$, equations (1-2) acquire the form:

$$\ddot{\varphi} + 2n_1\dot{\varphi} + (\lambda^2 - \ddot{y})\varphi = A\cos\omega t \quad (3)$$

$$\ddot{y} + 2n_2\dot{y} + q^2 y - (\varphi\ddot{\varphi} + \dot{\varphi}^2) = 0 \quad (4)$$

The solution by the method of multiple scales is obtained in the explicit analytical form. The formulas suggest that in the given excitation conditions the system performs oscillations at different frequencies, first of those being equal to the excitation frequency, while the second and the third ones being in two and three times larger.

Furthermore, the expressions for external force work and for work of dissipation forces were obtained. As follows from these formulas, despite of strongly nonlinear behavior of this mechanical system, the statements in [1-2] regarding its anomalously high efficiency are invalid.

Just as an illustrative example, the dependence of the external work per cycle upon excitation frequency is shown in Figure 3 for $l = 1$ sm, $m = 1$ kg, $\lambda = 1$ 1/s, $q = 2$ 1/s, $n_1 = 0.2$ 1/s, $n_2 = 0.2$ 1/s, $A = 0.1$ rad/s^2.

As is seen, maximum of the work is attained at the excitation frequencies, which slightly differ from the eigenfrequency of the pendulum. There is a good agreement between the analytical solution and the results of direct numerical integration of equations (1-2) in the time domain.

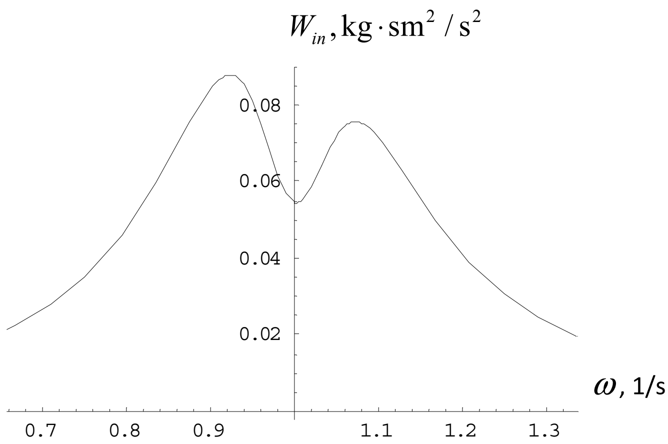

Figure 3.

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References

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